BISTABLE STRUCTURES EXHIBITING SNAP-THROUGH INSTABILITY: FROM SLAP BRACELETS TO THE VENUS FLYTRAP

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INTRODUCTION

Bistable structures, exemplified by the Venus flytrap [1] and slap bracelets (see Fig. 1), can transform quickly from one functional shape to the other upon mechanical actuation. Potential applications can be found in mechanical/electromechanical devices from bio-inspired robots to deployable aerocraft wings. Related challenges emerged include theoretically modeling the spontaneous curving and buckling of thin objects such as leaves, flowers, nanohelices, nanoscrolls and flexible electronics [2, 3]. Despite the significant modeling efforts about such large deformation of shell structures [4, 5], the nonlinear geometric effects remain poorly understood. Here we present a continuum elasticity theory that incorporates geometric nonlinearity for large deformation of shells, and investigates, through both theoretical analysis and table-top experiments, the geometric and mechanical conditions for bistability, and the role of edge effects. Our work classifies the conditions for bistability, defines the design space for bistable morphing structures, and extends the theory of plates and shells with large deformation. A mechanical framework is provided for analyzing morphogenesis associated with growth and instability, which will also facilitate the design of multistable structures, from bio-inspired robots to deployable structures in aerospace applications.

MATERIALS AND METHODS

Two pieces of thin latex rubber sheets are pre-stretched and bonded to a thicker acrylic strip. Upon releasing, the bonded system, driven by misfit strains, will deform either into a saddle shape, or one of two nearly cylindrical shapes (see Fig. 2). Bifurcation occurs when the strip is wide and thin enough.

RESULTS AND DISCUSSION

We develop a theoretical framework, using differential geometry, elasticity theory and variational principles, that quantitatively addresses large deformation of shell structures with geometric nonlinearity. We consider the deformation of an initially flat plate onto the surface of a torus or ellipsoid with two different principal curvatures along perpendicular directions. The strain tensor involved is given by differential geometry. The energy cost includes both bending energy and the in-plane stretching energy, while the surface stress serves as the driving force [2]. The equilibrium shape is achieved by applying variational principles to minimize the total potential energy.

Applying this theory naturally yields the geometric and mechanical conditions for bistability due to nonlinear geometric effects. Specifically, two dimensionless parameters are identified (one related to the dimensions and curvatures, and the other associated with the mechanical stresses) that control bistability.

The first parameter is the width ($W$) divided by the geometric mean (i.e., square root) of the thickness ($H$) and the radius of curvature (the smaller of the two principal radii of curvatures). The geometric condition states that this parameter has to go beyond a threshold in order for the system to exhibit bistability. We show that the ratio between the in-plane stretching energy and the bending energy goes as the 4th power of this geometric parameter, which explains the experimental observation (see Fig. 2) that only when the plate is wide and thin enough does bistability start to manifest itself, and the system prefers to assume a nearly cylindrical shape (except near the edges where it remains doubly curved, as shown by the dotted region in Fig. 2B and 2D). That is because when the stretching term is dominating (i.e., when the geometric parameter becomes large enough), the system
can reduce the total potential energy by taking a shape with nearly zero Gauss curvature.

The second parameter is related to the nature of the driving force (which is the surface stress in our model), i.e., the principal driving forces in two perpendicular directions have to bear different signs. We can show quantitatively that only when the second mechanical condition is also satisfied does the total energy profile exhibit double wells, i.e., there exist two locally minimum energy states.

Moreover, we reveal the importance of edge effects in determining the energetic preferences between two locally stable states for the case of finite plates. More specifically, the coiled state along the long axis as shown in Fig. 2C is more stable than the other state in Fig. 2A, because of the maximized edge area where the total energy is smaller than the interior area with zero Gauss curvature, due to the edge effect. This is consistent with the intuition that the coiled shape of a slap bracelet (Fig. 1F) is more stable than the uncoiled one (Fig. 1E)—it takes a lot more work to uncoil it than the other way around.

It is also worth pointing out that although the table-top experiments conducted are at the macroscopic scale, the mechanical principles addressed here should operate both at the macro- and micro-scales. In fact, one of the inspirations for us to consider the surface stresses as the driving forces in our theoretical models (for both our current work and the previous work on helical structures [2]) is that, often the spontaneous bending of structures gets manifested at the micro-/nanoscales where the surface-to-volume ratio becomes very significant.

Furthermore, we extend our study of bistable behaviors of shells to the Venus flytrap and address the mechanics behind the rapid motion of Darwin’s “most wonderful plant in the world”. A flytrap-robot is also designed with principles learnt from the Venus flytrap.

CONCLUSIONS

We establish a theoretical framework for large deformation of plates and shells that incorporates geometric nonlinearity, which is applied to study bistable structures. Our work classifies both geometric and mechanical conditions for bistability, and defines the design space for bistable morphing structures. Theoretical predictions are validated through tabletop experiments. This work also advances the understanding of plates and shells with large deformation, as well as the mathematical concept of developable and non-developable surfaces in the physical world. The results of our work will also promote understanding of morphogenesis in a variety of natural and engineered systems and facilitate the development of smart biomimetic devices as sensors, actuators, energy harvesting devices, artificial muscles, and bio-inspired robots.

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